# كليةٌ الرشُبي الجامععة قنسم هندسـة تّفّنـات الحاسوب 

## المرحلة الثانية

مادة اسس الاتصالات
(9) المحاضرة

## Systems

- A continuous-time (discrete-time) system H is an operator that transfer an input $x(t)(x[n])$ into output signal $\mathrm{y}(\mathrm{t})(\mathrm{y}[\mathrm{n}])$.



## Causal and Non-causal Systems

*A system is called causal if its output $\boldsymbol{y}(\boldsymbol{t})$ at an arbitrary time $\boldsymbol{t}$ depends on only the input $\boldsymbol{x}(\boldsymbol{t})$ for $\boldsymbol{t}$ lt $\mathbf{o}$.
*That is, the output of a causal system at the present time depends on only the present and/or past values of the input, not on its future values.
*Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system. A system is called non-causal if it is not causal.

## Examples 1

For the following check if the system is causal or non-causal

1. $Y[n]=x[n]-x[n-1]$
2. $Y[n]=a x[n]$
3. $Y[n]=x[2 n]$

## Answer

1. $Y[n]=x[n]-x[n-1]$
the system is causal because it depend on current and past value
2. $\mathrm{Y}[\mathrm{n}]=\mathrm{ax}[\mathrm{n}]$
the system is causal because it depend on current value
3. $Y[n]=x[2 n]$
the system is non-causal because it depend on future value

## Time-Invariant and Time- variant Systems

A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal.

## Steps of solution

Step1// find $y[n, k]$
That mean replace each $x[n]$ and put $x[n-k]$

Step2// find $y[n-k]$
That mean replace each $n$ and put $n-k$

Step 3
If $y[n, k]=y[n-k]$ the system is invariant

## Example 2

Check if the system is variant or invariant

$$
\mathrm{Y}[\mathrm{n}]=\mathrm{x}[\mathrm{n}]-\mathrm{x}[\mathrm{n}-1]
$$

## solution

$$
Y[n]=x[n]-x[n-1]
$$

Step1// find $\mathrm{y}[\mathrm{n}, \mathrm{k}]$

$$
Y[n, k]=x[n-k]-x[n-k-1]
$$

Step2// find $\mathrm{y}[\mathrm{n}-\mathrm{k}]$

$$
y[n-k]=x[n-k]-x[n-k-1]
$$

$y[n, k]=y[n-k] \rightarrow$ the system is invariant

## Example 3

Check if the system is variant or invariant

$$
Y[n]=n x[n]
$$

- Answer

Step1// find $\mathrm{y}[\mathrm{n}, \mathrm{k}]$

$$
\mathrm{Y}[\mathrm{n}, \mathrm{k}]=\mathrm{n} \times[\mathrm{n}-\mathrm{k}]
$$

Step2// find $\mathrm{y}[\mathrm{n}-\mathrm{k}]$

$$
y[n-k]=[n-k] x[n-k]
$$

$\mathrm{y}[\mathrm{n}, \mathrm{k}] \neq \mathrm{y}[\mathrm{n}-\mathrm{k}], \rightarrow$ the system is variant

## Linear and non-linear system

A linear system is any system that obeys the properties of:
Scaling (homogeneity)
A system $H$ has the input signal $f(t)$ and scaling factor $k$ then
$H(k f(t))=k H(f(t))$
Superposition (additively)
A system $\mathbf{h}$ has the input signals $\times 1 \& x 2$ then the output signal must be $y 1+y 2=x 1+x 2$

## How to check the system is linear or not

- step of solution

1. Determine $Y_{1}$, every $x[n]$ equal to $x_{1}[n]$
2. Determine $Y_{2}$, every $x[n]$ equal to $x_{2}[n]$
3. Determine $Y_{3}=a_{1} Y_{1}+a_{2} Y_{2}$
4. Determine $\mathrm{Y} 4=R\left[\mathrm{a}_{1} \mathrm{x}_{1}+\mathrm{a}_{2} \mathrm{x}_{2}\right]$
5. If $\mathrm{Y} 3=\mathrm{Y} 4$ the system is linear

## Example 4

- Determine if the system $\mathrm{Y}[\mathrm{n}]=\mathrm{nX}[\mathrm{n}]$, is linear or not


## Solution

Step1 find $Y_{1}[n]$

$$
\mathrm{Y}_{1}[\mathrm{n}]=\mathrm{nX} \mathrm{X}_{1}[\mathrm{n}]
$$

Step2 Find $\mathrm{Y}_{2}[\mathrm{n}]$

$$
\mathrm{Y}_{2}[\mathrm{n}]=\mathrm{nX}_{2}[\mathrm{n}]
$$

- Step3 Find $\mathrm{Y}_{3}$

$$
\begin{gathered}
Y 3=a_{1} \mathbf{Y}_{1}+a_{2} \mathbf{Y}_{2} \\
Y 3=a_{1} n X_{1}[n]+a_{2} n X_{2}[n]
\end{gathered}
$$

- Step4 Find $\mathbf{Y}_{4}$

$$
\begin{aligned}
& \mathrm{Y}[\mathrm{n}]=\mathrm{nX}[\mathrm{n}] \\
& \mathrm{Y} 4=R\left[\mathrm{a}_{1} \mathrm{x}_{1}+\mathrm{a}_{2} \mathrm{x}_{2}\right] \\
& \mathrm{Y} 4=n\left[\mathrm{a}_{1} \mathrm{x}_{1}[\mathrm{n}]+\mathrm{a}_{2} \mathrm{x}_{2}[\mathrm{n}]\right] \\
& \mathrm{Y} 4=\mathrm{a}_{1} \mathrm{n} \mathrm{x}_{1}[\mathrm{n}]+\mathrm{a}_{2} \mathrm{n} \mathrm{x}_{2}[\mathrm{n}]
\end{aligned}
$$

$\mathrm{Y} 4=\mathrm{Y} 3 \Rightarrow$ the system is linear

## Example 5

Determine if the system $\mathrm{Y}[\mathrm{n}]=e^{x[n]}$ is linear or not Solution
Step1 find $Y_{1}[n]$

$$
\mathrm{Y}_{1}[\mathrm{n}]=e^{x 1[n]}
$$

Step2 Find $\mathrm{Y}_{2}[\mathrm{n}]$

$$
\mathrm{Y}_{2}[\mathrm{n}]=e^{x 2[n]}
$$

- Step3 Find $\mathbf{Y}_{3}$

$$
\begin{aligned}
& \mathbf{Y} 3=\mathbf{a}_{\mathbf{1}} \mathbf{Y}_{\mathbf{1}}+\mathbf{a}_{\mathbf{2}} \mathbf{Y}_{\mathbf{2}} \\
& \mathbf{Y} 3=\mathbf{a}_{1} e^{x 1[n]}+\mathbf{a}_{\mathbf{2}} e^{x 2[n]}
\end{aligned}
$$

- Step4 Find $\mathbf{Y}_{4}$

$$
\begin{aligned}
& \mathrm{Y} 4=R\left[\mathrm{a}_{1} \mathrm{x}_{1}+\mathrm{a}_{2} \mathrm{x}_{2}\right] \\
& \mathrm{Y}[\mathrm{n}]=e^{x[n]} \\
& \mathrm{Y} 4=e^{a 1 x 1[n]+a 2 x 2[n]}
\end{aligned}
$$

$\mathrm{Y} 4 \neq \mathrm{Y} 3 \quad \rightarrow$ the system is non linear

