Sampling theorem

The **sampling theorem** specifies the minimum-sampling rate at which a continuous-time signal needs to be uniformly sampled so that the original signal can be completely recovered or reconstructed by these samples alone.

This is usually referred to as Shannon's sampling theorem in the literature.

$$f_s \geq 2f_m$$

Where

f_s represent sampling frequency ,

f_m represent the message frequency

$$\mathbf{T = \frac{1}{Fs}}$$

$$\mathbf{F_s = \frac{1}{T}}$$

$$\boldsymbol{\omega_m = 2\pi f_m}$$

<u>Example 1</u>: The sampling frequency of a signal is Fs = 2000 samples per second. Find its Nyquist interval.

Answer:

Given Fs = 2000 samples per second

Nyquist interval, $T = \frac{1}{Fs} = \frac{1}{2000} = 0.5$ msec.

<u>Example 2</u>: Find the Nyquist rate and Nyquist interval for the signal

$f(t) = 1 + \text{sinc}\ \mathbf{300\pi}\ t.$

Answer:

Frequency, $\omega_m = 300\pi$

$2\pi f_m = 300\pi$

$2f_m = 300$ Hz

Nyquist rate, $Fs = 2f_m = 300$ Hz

Nyquist interval, $T = \frac{1}{2fm} = \frac{1}{300} = 3.3$ msec

Example 3: Find the Nyquist rate and Nyquist interval of sin ($2\pi t$).

Answer

Here $\omega_m = 2\pi$

But $\omega_m = 2\pi f_m$

$2\pi f_m = 2\pi$

$\therefore f_m = 1$ Hz

Nyquist rate, $Fs = 2f_m = 2$ Hz

Nyquist interval, $T = \frac{1}{2fm} = \frac{1}{2} = 0.5$ sec

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Example 4: Determine the Nyquist rate of the signal $x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$.

Answer

$$x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$$

Highest frequency component in 1 is zero

Highest frequency component in $\cos 2000\pi t$ is $\omega m1 = 2000\pi$

Highest frequency component in $\sin 4000\pi t$ is $\omega m2 = 4000\pi$

So the maximum frequency component in x(t) is $\omega m = 4000\pi$

$\therefore 2\pi fm = 4000\pi$

$2fm = 4000$

Nyquist rate, $Fs = 2fm = 4000$ Hz.